



**YEAR 12
MATHEMATICS
SPECIALIST**

**Test 4, 2023
Section Two: Calculator Allowed
Motion and Statistics**

STUDENT'S NAME: _____

DATE: Thursday 7th September

TIME: 50 minutes

MARKS: 48
ASSESSMENT %: 10

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: 1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

This page has been left intentionally blank.

Question 1**(8 marks)**

A particle is traveling in a straight line with an initial velocity of 3m/s and an acceleration of $a(t) = 9e^{3t}$.

(a) Determine the distance travelled in the first 3 seconds. (4 marks)

(b) Determine an expression for acceleration in terms of displacement, $a(x)$. (2 marks)

c) Hence, or otherwise, determine the starting position of the object if, at the 3 second point, the magnitude of the displacement is half the magnitude of the acceleration. (2 marks)

Question 2**(8 marks)**

Researchers found the reaction time of mice by playing a loud buzzing sound which stopped when the mouse turned around. The reaction times were said to be normally distributed with an estimated mean of 151 milliseconds and a standard deviation of σ milliseconds. Samples of 40 mouse's reaction times are recorded and the means are found to create the sample mean distribution \bar{X} .

(a) State the distribution for \bar{X} and its parameters. (3 marks)

(b) Discuss the effects that increasing the sample size would have on \bar{X} . (1 mark)

After many samples had been gathered, it was noted that 75% of the samples had a total duration of buzzing of 6000 milliseconds or greater.

(c) Calculate σ correct to 2 decimal places. (3 marks)

A researcher is tasked with calculating a confidence interval to capture the true population mean. The researcher stated: "I am going to use a very small sample size for my confidence interval as that will increase its width and hence increase my chances of capturing the population mean".

(d) Explain an issue with the researcher's comment. (1 mark)

Question 3**(11 marks)**

A particle is travelling in a straight line under simple harmonic motion. It is observed only when it passes through the point $x = 0$ periodically at $t = 1, 3, 5, 7, 9, \dots$ seconds.

- (a) Discuss what the viewer would observe about the particle's velocity each time it is seen at the point $x = 0$. (2 marks)

- (b) Explain why the function $x(t) = A \cos\left(\frac{\pi}{2}t\right)$, $A \in \mathbb{R}$ should be used to model this particle's motion. A brief sketch can be used. (2 marks)

It is first observed with a velocity of 10m/s .

- (c) Determine an expression for the particle's displacement in terms of time, $x(t)$. (2 marks)

(d) Prove that the object undergoes simple harmonic motion. (2 marks)

(e) Determine the percentage of time this particle spent travelling faster than $6m/s$. (3 marks)

Question 4

(13 marks)

In a court of law, a tomato sauce company, NATRAL, is being sued for the inclusion of too much salt within their product. NATRAL claims that their tomato sauce is naturally created and so the salt content may vary per bottle, but they promise an average of 4.50g per bottle.

Lawyer 1, the lawyer suing NATRAL, sampled 100 bottles randomly, with a mean of 4.888g and a sample standard deviation of 0.1728g.

- (a) Comment, by creating and using a 99.9% confidence interval, on the promise made by NATRAL. (4 marks)

- (b) Calculate the sample size required to reduce the 99.9% confidence interval's width to below 0.06. (2 marks)

Lawyer 2, the lawyer in charge of defending NATRAL, comes up with a cunning plan. This lawyer took the original sample of 100 bottles and split it into two smaller samples of 50.

Table 1

	Sample's Mean	Sample Standard Deviation	Lower limit of confidence interval	Higher Limit of confidence interval	Confidence level
Sample A, size 50	<i>a</i>	0.0756g	4.7180g	4.7600g	<i>b</i>
Sample B, size 50	5.034g	<i>c</i>	5.0074g	5.0606g	95%

- (c) Determine the values of a , b and c . (1, 2, 2 = 5 marks)

This lawyer wanted to discredit the whole lawsuit by stating “These 100 bottles could not be from NATRAL as these two samples seem to be taken from different sources”.

- (d) Discuss this statement with the use of the given confidence intervals from *Table 1*. (1 mark)

It was later found that Lawyer 2 chose sample A by selecting the lowest 50 scores from the original sample and chose sample B by selecting the highest 50 scores from the original sample.

- (e) Explain the problem in this methodology. (1 mark)

Question 5**(8 marks)**

An object is traveling in a straight line. It was initially observed passing through the origin with a velocity of 4km/s. This object's velocity is always greater than 0 km/s.

The object's acceleration is defined as $a(x) = x + 4$

- (a) Show that the velocity can be defined as $v(x) = x + 4$ (3 marks)

- (b) Hence, or otherwise, determine an expression for the object's displacement in terms of time, $x(t)$. (3 marks)

For this experiment, an object is seen to be stationary when it is travelling slower than $0.000\ 000\ 01\text{km/s}$ [1×10^{-8}]

- (c) Determine when and from where the object likely started moving. (2 marks)
(Hint: Do not round your working before the final answer)

END OF QUESTIONS